Unitarisation of EFT Amplitudes for Dark Matter Searches at the LHC

Michael A. Schmidt

10 July 2017

Dark Side of the Universe

based on

N. Bell, G. Busoni, A. Kobakhidze, D. Long, MS

JHEP 1608 (2016) 125 [1606.02722 [hep-ph]]

THE UNIVERSITY OF SYDNEY

CoEPP

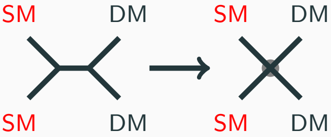
ARC Centre of Excellence for Particle Physics at the Terascale

Hunting Dark Matter at the Large Hadron Collider (run 1)

Economical Effective Field Theory

- momentum transfer $Q \ll M$

$$\frac{g_\chi g_q}{Q^2 - M^2} \Rightarrow -\frac{g_\chi g_q}{M^2} \equiv \frac{1}{M_*^2}$$



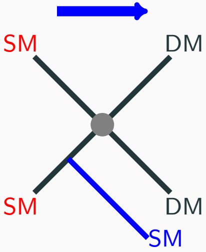
- use EFT operators

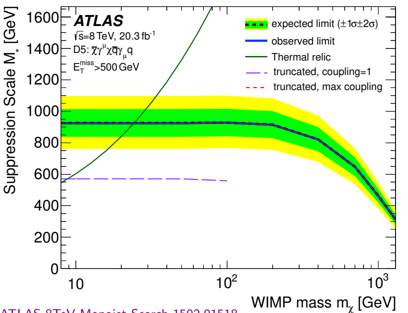
$$\mathcal{L}_{\mathrm{EFT}} = \frac{1}{M_*^2}(\bar{q}\Gamma_q q)(\bar{\chi}\Gamma_\chi \chi)$$

$$\Gamma_i \in \left\{1, \gamma^5, \gamma^\mu, \gamma^5\gamma^\mu, \sigma^{\mu\nu}\right\}$$

- operator coefficients are independent variables

Mono-j,W,Z,γ,H search





ATLAS 8TeV Monojet Search 1502.01518

Issues of EFT

Scattering amplitudes with higher-dimensional operators (with $d > 4$) grow indefinitely

e.g. Dimension-6 operator

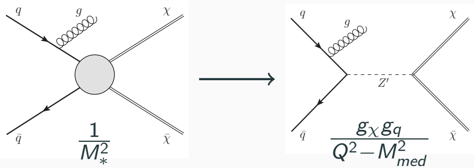
$$\mathcal{A}(s) \simeq \frac{s}{M_*^2} \xrightarrow{s\to\infty} \infty$$

$\Rightarrow$ Violation of perturbative unitarity

Monojet searches: Shoemaker, Vecchi 1112.5457; Endo, Yamamoto 1403.6610; Yamamoto 1409.5775; El-Hedri, Shepherd, Walker 1412.5660

Solutions

Simplified Models



✗ abandon EFT

✗ model-dependent

✓ perturbative unitarity

Truncation

EFT is expansion in $\frac{Q^2}{M^2}$

$$\frac{1}{Q^2 - M^2} = -\frac{1}{M^2}\left[1 + \frac{Q^2}{M^2} + \mathcal{O}\left(\frac{Q^4}{M^4}\right)\right]$$

Discard events if

$$Q > M \equiv \frac{M_*}{\sqrt{g_q g_\chi}}$$

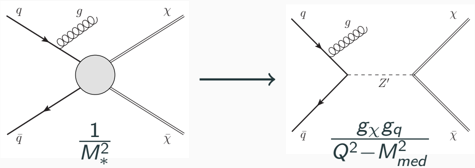
✓ retain EFT

✗ highest energy events discarded

Busoni, De Simone, Morgante, Riotto 1307.2253, +Gramling 1402.1275, +Jacques 1405.3101

Solutions

Simplified Models



- ✗ abandon EFT
- ✗ model-dependent
- ? perturbative unitarity unless oversimplified

Truncation

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- ✓ retain EFT
- ✗ still model-dependent
- ✗ highest energy events discarded

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Have a closer look at unitarity of S-matrix . . .

Unitarity of S-Matrix

Scattering processes described by S matrix

$$S = \mathbb{I} + 2i\,T$$

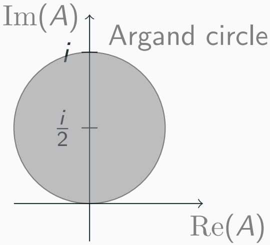
S-matrix is unitary

$$S^\dagger S = \mathbb{I}$$

For an eigenvalue A of T

$$|1 + 2iA|^2 = 1$$

$$\Rightarrow \left|A - \frac{i}{2}\right| = \frac{1}{2}$$



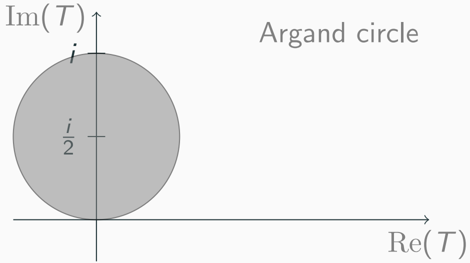
In terms of T-matrix unitarity implies the optical theorem

$$T - T^\dagger = 2i\,T^\dagger T$$

Unitarity and the K-Matrix

- Perturbative expansion of S-matrix not unitary at fixed order

$$S = \mathbb{I} + 2i\,T \qquad\qquad T = T_1 + T_2 + T_3 + \ldots$$



Optical theorem

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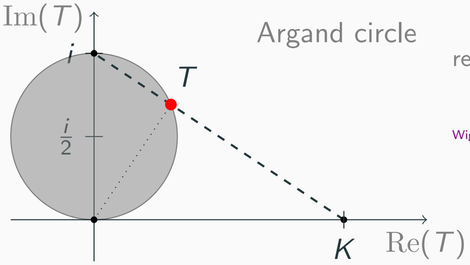
$$S = \mathbb{I} + 2i\,T \qquad\qquad T = T_1 + T_2 + T_3 + \ldots$$

- Expansion of K-matrix unitary order by order Heitler 1941; Schwinger 1948

$$S = \frac{\mathbb{I} + i\,K}{\mathbb{I} - i\,K} \qquad\qquad K = K_1 + K_2 + K_3 + \ldots$$

S is Cayley transform of K: S unitary $\Leftrightarrow$ K hermitean

S time-reversal invariant $\Leftrightarrow$ K symmetric and thus real



Optical theorem

$$T - T^\dagger = 2i\,T^\dagger T$$

rewrite to

$$\left(T^{-1} + i\,\mathbb{I}\right)^\dagger = T^{-1} + i\,\mathbb{I} \equiv K^{-1}$$

Wigner 1946; Wigner, Eisenbud 1947; Gupta 1950

$$\Rightarrow T = \frac{1}{K^{-1} - i\,\mathbb{I}}$$

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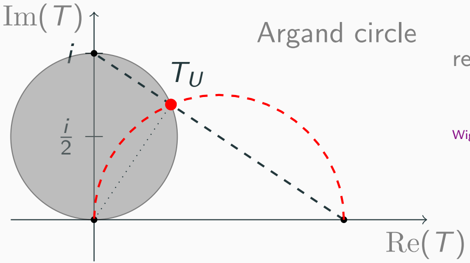

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Wigner 1946; Wigner, Eisenbud 1947; Gupta 1950; Kilian et. al 1408.6207

$$\Rightarrow T_U = \frac{1}{\mathrm{Re}\,(T^{-1}) - i\,\mathbb{I}}$$

K-Matrix Unitarisation

$$T_U = \frac{1}{\mathrm{Re}\left(T^{-1}\right) - i\,\mathbb{I}}$$

Wigner 1946; Wigner, Eisenbud 1947; Gupta 1950; Kilian et. al 1408.6207

- Well-known for WW-scattering e.g. Alboteanu et. al 0806.4145; Kilian et. al 1408.6207 and hadronic physics e.g. Chung et. al 1995
- Other unitarisation methods: e.g. Padé, Inverse Amplitude, N/D, ...
- K-matrix unitarisation is "minimal": no new resonances introduced by unitarisation

! Does not describe resonances of true high energy theory

→ Resonances can be added by hand, if necessary

- Scattering amplitudes well behaved at high energies

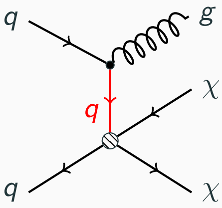
→ Allows to derive meaningful limits on EFT models from LHC collisions with high centre of mass energies

Simplifying Assumptions: Collinear Factorisation in EFT

- Apply K-matrix formalism to monojet searches
- Amplitudes of t-channel processes have soft collinear singularity

$\Rightarrow$ Dominated by quarks emitted in direction of gluon

- t-channel quark almost on-shell



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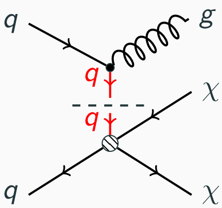
$\Rightarrow$ Amplitude and cross section factorise



$$\sigma_{q\bar{q}\to j\bar{\chi}\chi}(s) = \sigma_{\bar{q}q\to\bar{\chi}\chi}(s(1-z))F_{q\to qg}(z,\theta)$$

Angle of gluon jet in CoM frame θ; Energy of gluon jet $E = \sqrt{\hat{s}}\frac{z}{2}$

Simplifying Assumptions: Possible States

220 neutral two-particle states with zero baryon and lepton number

Assumptions

- Only singlet color state

$$\frac{R\bar{R} + V\bar{V} + B\bar{B}}{\sqrt{3}}$$

- Only one flavour state:

$$q\bar{q} \equiv \frac{u\bar{u} + d\bar{d} + s\bar{s} + c\bar{c} + b\bar{b} + t\bar{t}}{\sqrt{6}} \ ,$$

- Neglect electroweak interactions: $q\bar{q}$ only couples to $\chi\bar{\chi}$

Unitarisation of Each Partial Wave

- Partial wave decomposition using two-particle helicity states in terms of Wigner D functions $D^J_{\lambda\lambda'}$ Jacob,Wick 1959

$$\langle \Omega \lambda_c \lambda_d | T | 0 \lambda_a \lambda_b \rangle = \frac{1}{4\pi} \sum_J (2J+1)\, T^J_{\lambda'\lambda} \mathcal{D}^{J*}_{\lambda\lambda'}(\phi, \theta, 0)$$

with partial waves $T^J_{\lambda'\lambda}$ of defined total angular momentum J

- Unitarity condition holds for each partial wave separately

$$T^J - T^{J\dagger} = 2i\, T^{J\dagger} T^J$$

- Unitarised partial wave

$$T^J_U \equiv \frac{1}{\mathrm{Re}\left[(T^J)^{-1}\right] - i\,\mathbb{I}}$$

Simple Two Channel Model: One Effective Operator

Effective Lagrangian: $\mathcal{L}_1 = \frac{1}{\Lambda_{q\chi}^2}\bar{q}\gamma_\mu P_R q \bar{\chi}\gamma^\mu P_L \chi$

$\rightarrow$ possible UV completion: coloured scalar t-channel mediator

For $s \gg m_\chi^2, m_q^2$, the T-matrix in basis of $(|q_R\bar{q}_L\rangle, |\chi_L\bar{\chi}_R\rangle)$

$$T = \begin{pmatrix} q_R\bar{q}_L \to q_R\bar{q}_L & \chi_L\bar{\chi}_R \to q_R\bar{q}_L \\ q_R\bar{q}_L \to \chi_L\bar{\chi}_R & \chi_L\bar{\chi}_R \to \chi_L\bar{\chi}_R \end{pmatrix} = -\frac{1}{16\pi^2}\frac{s}{\Lambda_{q\chi}^2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\sin^2\frac{\theta}{2}$$

Partial wave decomposition: only $J = 1$ $[D^1_{-1,1} = D^1_{1,-1} = \sin^2\frac{\theta}{2}]$

$$T^1 = -\frac{1}{12\pi}\frac{s}{\Lambda_{q\chi}^2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Unitarised T-matrix $T_U^J \equiv \frac{1}{\mathrm{Re}\left[(T^J)^{-1}\right] - i\,\mathbb{I}}$

$$T_U^1 = \frac{1}{s^2 + 144\pi^2\Lambda_{q\chi}^4}\begin{pmatrix} is^2 & -12\pi s\Lambda_{q\chi}^2 \\ -12\pi s\Lambda_{q\chi}^2 & is^2 \end{pmatrix} \xrightarrow{s\to\infty} i\,\mathbb{I}$$

Effective Operator D5

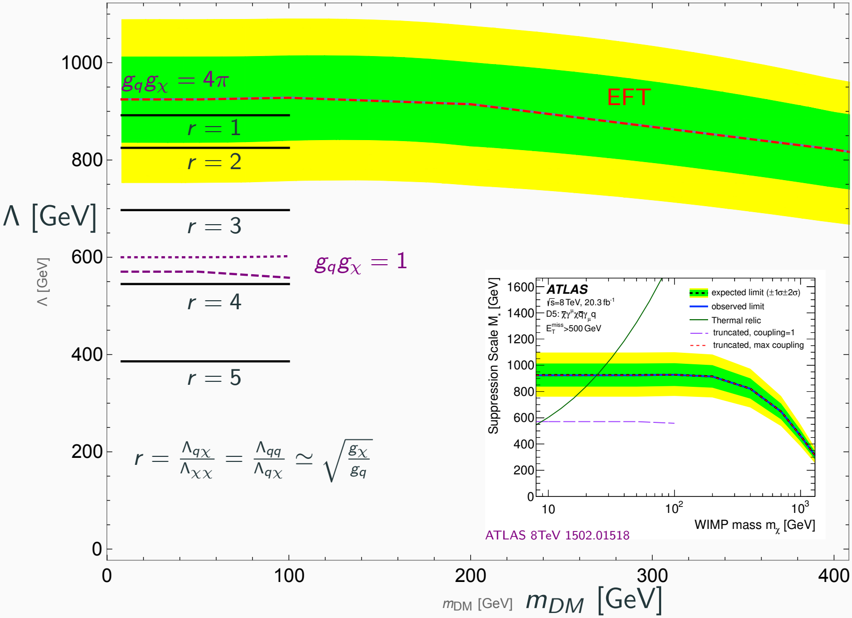
$$\mathcal{L}_{D5} = \frac{1}{2\Lambda_{qq}^2}\bar{q}\gamma_\mu q\bar{q}\gamma^\mu q + \frac{1}{\Lambda_{q\chi}^2}\bar{q}\gamma_\mu q\bar{\chi}\gamma^\mu\chi + \frac{1}{2\Lambda_{\chi\chi}^2}\bar{\chi}\gamma_\mu\chi\bar{\chi}\gamma^\mu\chi \, .$$

- Additional operators with four quarks q or DM particles χ
- Simplifying assumption motivated by s-channel Z' exchange

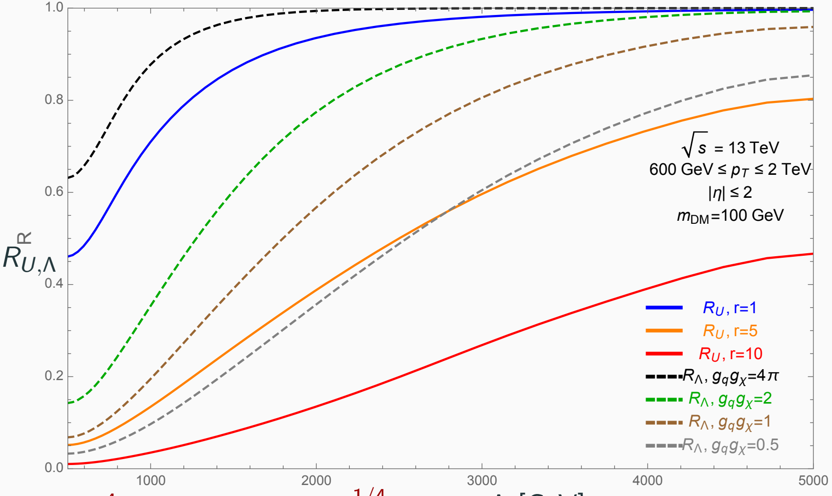
$$r = \frac{\Lambda_{q\chi}}{\Lambda_{\chi\chi}} = \frac{\Lambda_{qq}}{\Lambda_{q\chi}}$$

- Same simplifications regarding two-particle states
- Collinear factorisation
- Include quark jets

ATLAS 8 TeV Monojet Search: D5 Operator ATLAS 1502.01518



13 TeV Monojet: Suppression Compared to EFT



$\sigma \propto \Lambda^{-4} \to \Lambda$ reduced by $R_U^{1/4}$ Λ [GeV]

$$R_U = \frac{\sigma_{\text{unit,coll.}}}{\sigma_{\text{EFT,coll.}}} \qquad R_\Lambda = \frac{\sigma_{\text{trunc,coll.}}}{\sigma_{\text{EFT,coll.}}} \qquad r = \frac{\Lambda_{q\chi}}{\Lambda_{\chi\chi}} = \frac{\Lambda_{qq}}{\Lambda_{q\chi}}$$

Conclusions and Outlook

- K-matrix formalism provides firm theoretical framework based on unitarity of S-matrix
- Model-independent
- Unitarised EFT is a new way to present LHC limits

What next?

- Go beyond collinear approximation
- Apply to other processes

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Thank you!

Backup Slides

Scattering amplitudes with higher-dimensional operators (with $d > 4$) grow indefinitely

e.g. Dimension-6 operator

$$\mathcal{A}(s) \simeq \frac{s}{M_*^2} \xrightarrow{s\to\infty} \infty$$

$\Rightarrow$ **Violation of perturbative unitarity**

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Issues of EFT

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If not using SMEFT, $SU(2)_L$ gauge invariance violated for $\xi \neq 1$

$$\frac{1}{M_*^2}\,\bar{\chi}\gamma_\mu\chi\,(\bar{u}\gamma^\mu u + \xi\bar{d}\gamma^\mu d)$$

$\Rightarrow$ Unitarity violated with $\xi \neq 1$ at high energies

Bell, Cai, Dent, Leane, Weiler 1503.07874

Simple Two Channel Model: Vector-Interactions

Effective Lagrangian

$$\mathcal{L} = \frac{1}{2\Lambda_{qq}^2}\bar{q}\gamma_\mu P_R q\bar{q}\gamma^\mu P_R q + \frac{1}{\Lambda_{q\chi}^2}\bar{q}\gamma_\mu P_R q\bar{\chi}\gamma^\mu P_R \chi + \frac{1}{2\Lambda_{\chi\chi}^2}\bar{\chi}\gamma_\mu P_R \chi\bar{\chi}\gamma^\mu P_R \chi$$

For $s \gg m_\chi^2, m_q^2$, the T-matrix in basis of $(|q_R\bar{q}_L\rangle, |\chi_L\bar{\chi}_R\rangle)$

$$T = \begin{pmatrix} q_R\bar{q}_L \to q_R\bar{q}_L & \chi_L\bar{\chi}_R \to q_R\bar{q}_L \\ q_R\bar{q}_L \to \chi_L\bar{\chi}_R & \chi_L\bar{\chi}_R \to \chi_L\bar{\chi}_R \end{pmatrix} = -\frac{1}{16\pi^2}\begin{pmatrix} \frac{2s}{\Lambda_{qq}^2} & \frac{s}{\Lambda_{q\chi}^2} \\ \frac{s}{\Lambda_{q\chi}^2} & \frac{2s}{\Lambda_{\chi\chi}^2} \end{pmatrix}\cos^2\frac{\theta}{2}$$

Partial wave decomposition: only $J = 1$ $\qquad D_{11}^1 = \cos^2\frac{\theta}{2}$

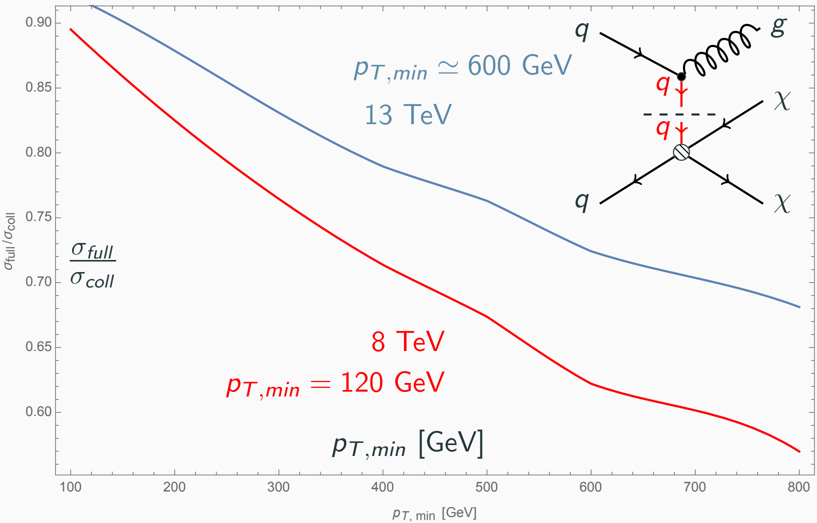
$$T^1 = -\frac{1}{12\pi}\begin{pmatrix} \frac{2s}{\Lambda_{qq}^2} & \frac{s}{\Lambda_{q\chi}^2} \\ \frac{s}{\Lambda_{q\chi}^2} & \frac{2s}{\Lambda_{\chi\chi}^2} \end{pmatrix}$$

Unitarised T-matrix $T_U^J \equiv \frac{1}{\mathrm{Re}[(T^J)^{-1}] - i\,\mathbb{I}}$ assuming $\Lambda_{q\chi}^2 = \Lambda_{qq}\Lambda_{\chi\chi}$

$$T_{U,r}^1 = \frac{1}{r^2s^2 - 8i\pi\left(r^4+1\right)s\Lambda_{q\chi}^2 - 48\pi^2 r^2\Lambda_{q\chi}^4}\begin{pmatrix} is^2r^2 + 8\pi s\Lambda_{q\chi}^2 & 4\pi r^2 s\Lambda_{q\chi}^2 \\ 4\pi r^2 s\Lambda_{q\chi}^2 & is^2r^2 + 8\pi s\Lambda_{q\chi}^2 \end{pmatrix}$$

$r \equiv \frac{\Lambda_{q\chi}}{\Lambda_{\chi\chi}} = \frac{\Lambda_{qq}}{\Lambda_{q\chi}}$

Validity of Collinear Factorisation



Agrees with Birkedal, Matchev, Perelstein hep-ph/0400304

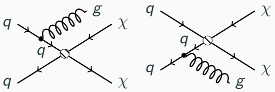
Similar effect for unitarised and EFT cross section expected

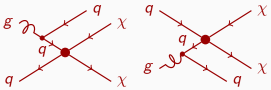
⇒ Partial cancellation in ratio ⇒ Collinear factorisation works well

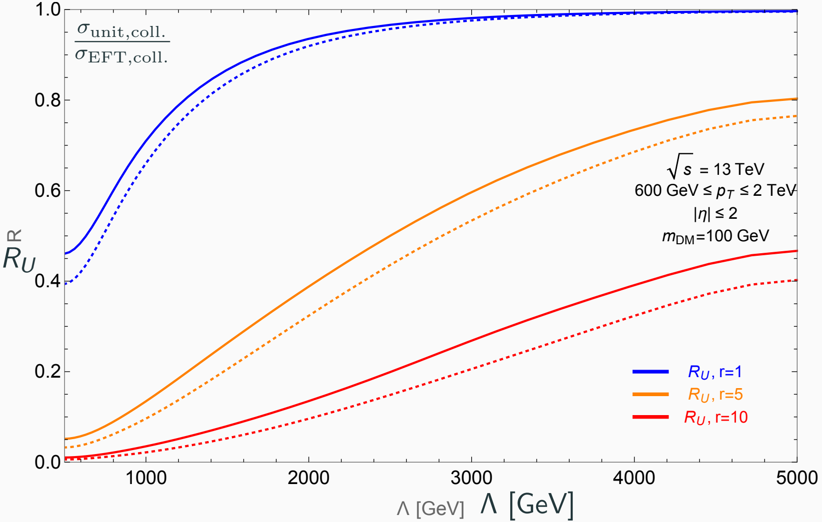
Quark Jets

$$R_U = \frac{\sigma_{\text{unit,coll.}}}{\sigma_{\text{EFT,coll.}}}$$

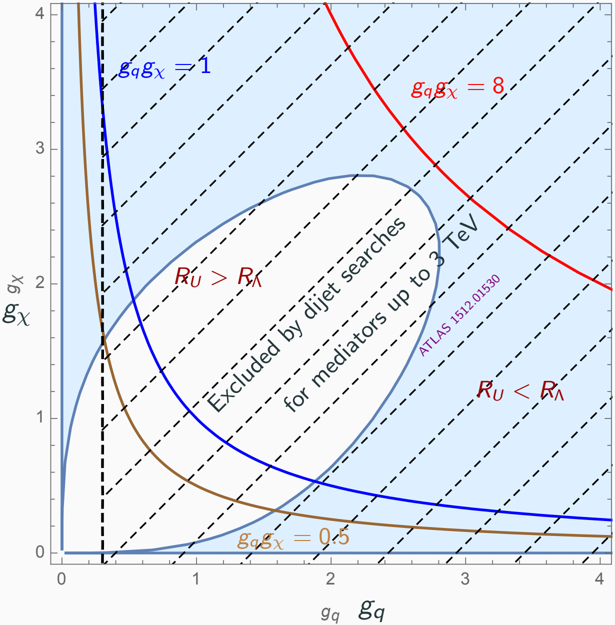
$$r = \frac{\Lambda_{q\chi}}{\Lambda_{\chi\chi}} = \frac{\Lambda_{qq}}{\Lambda_{q\chi}}$$







Truncation vs. Unitarisation



- EFT cutoff scale

$$\frac{1}{\Lambda^2} = \frac{g_\chi g_q}{M^2}$$

- Ratios

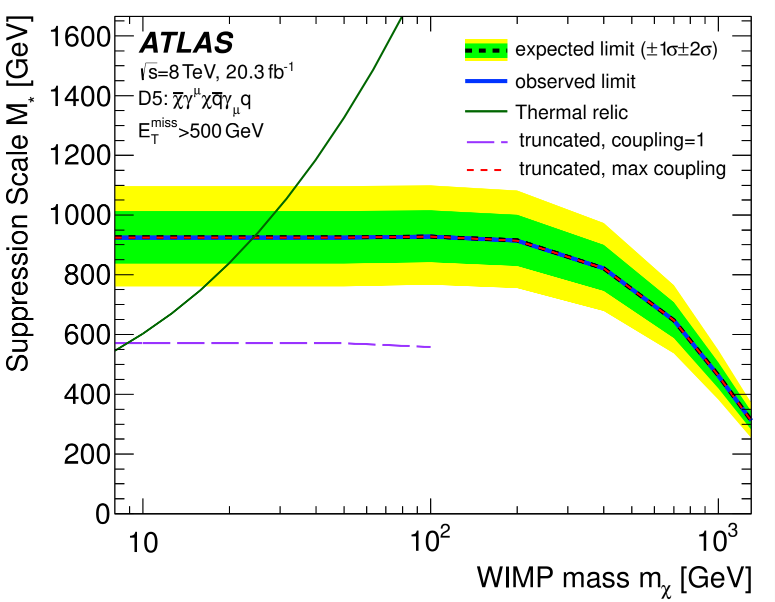
$$R_U = \frac{\sigma_{\text{unit,coll.}}}{\sigma_{\text{EFT,coll.}}}$$

$$R_\Lambda = \frac{\sigma_{\text{trunc,coll.}}}{\sigma_{\text{EFT,coll.}}}$$

$$r \simeq \sqrt{\frac{g_\chi}{g_q}}$$

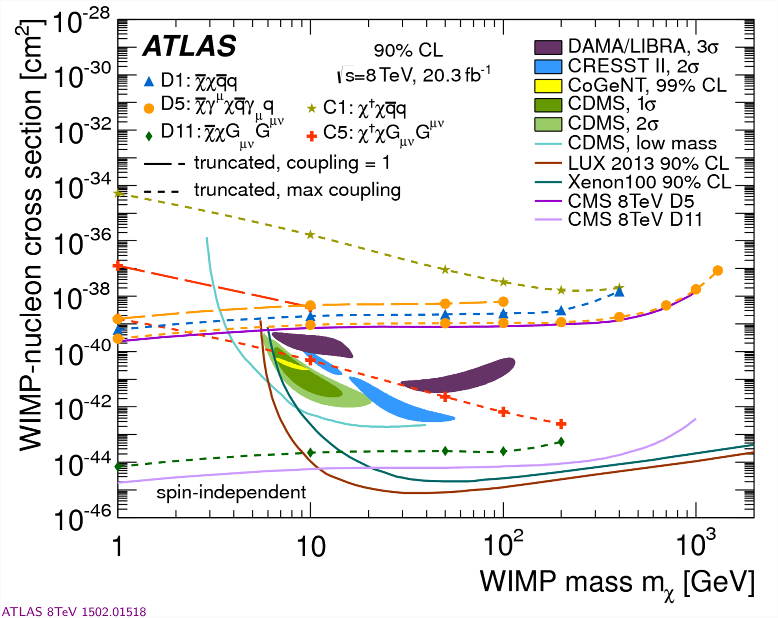
- LHC dijet search important

ATLAS 8 TeV Monojet Search at LHC Run 1: 8TeV



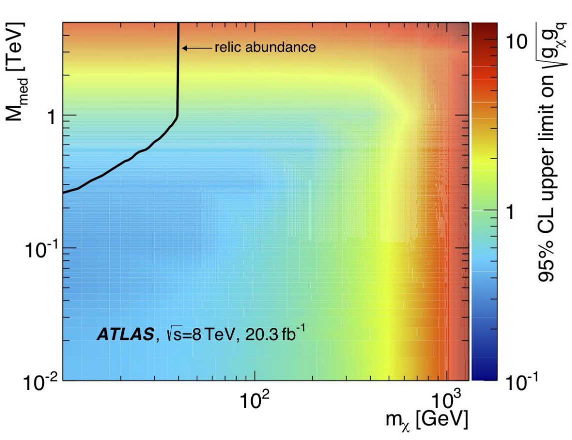
ATLAS 8TeV 1502.01518

Comparison to Direct Detection

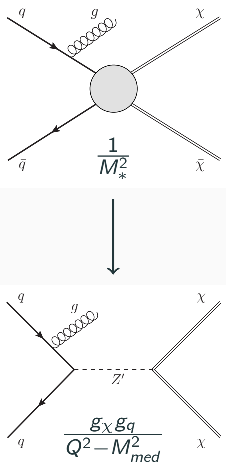


ATLAS 8TeV 1502.01518

Solutions: Simplified Models



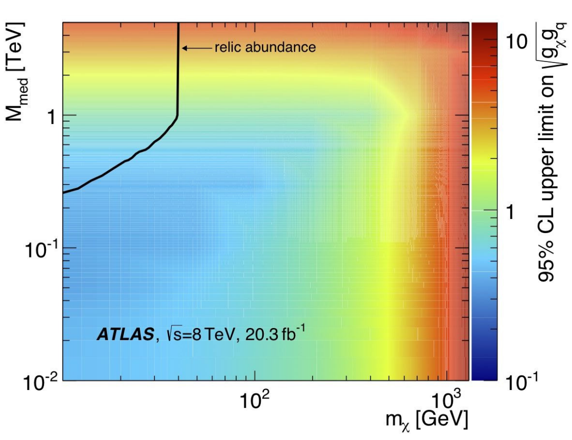
ATLAS 8TeV 1502.01518



✓ perturbative unitarity

✗ model-dependent

Solutions: Simplified Models



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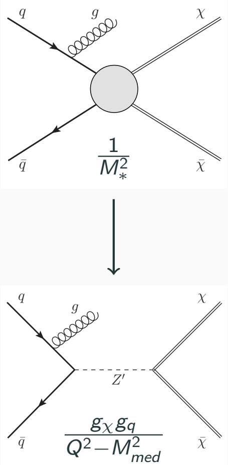


? perturbative unitarity not solved for all simplified models

✗ model-dependent

Solutions: Truncation

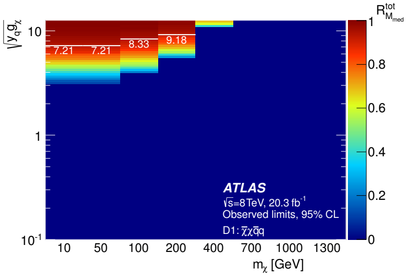
Effective field theory: Expansion in $\frac{Q^2}{M^2}$

$$\frac{1}{Q^2 - M^2} = -\frac{1}{M^2}\left[1 + \frac{Q^2}{M^2} + \mathcal{O}\left(\frac{Q^4_{tr}}{M^4}\right)\right]$$

Discard events if Busoni, De Simone, Morgante, Riotto 1307.2253, +Gramling 1402.1275, +Jacques 1405.3101

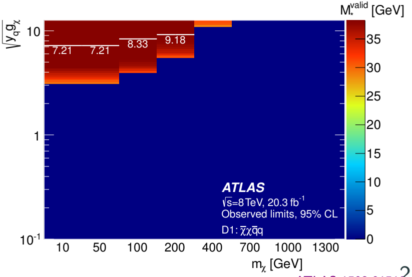
$$Q > M \equiv \frac{M_*}{\sqrt{g_q g_\chi}}$$

valid events



ATLAS 1502.01518

truncated limits



ATLAS 1502.01518

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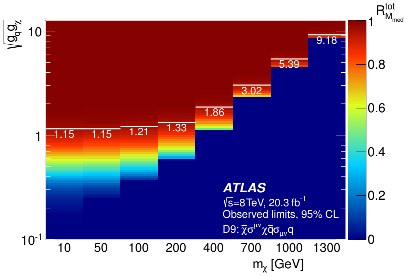
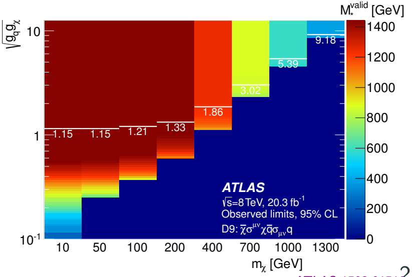
$$Q > M \equiv \frac{M_*}{\sqrt{g_q g_\chi}}$$

valid events



ATLAS 1502.01518

truncated limits



ATLAS 1502.01518

Solutions: Truncation

Effective field theory: Expansion in $\frac{Q^2}{M^2}$

$$\frac{1}{Q^2 - M^2} = -\frac{1}{M^2}\left[1 + \frac{Q^2}{M^2} + \mathcal{O}\left(\frac{Q_{tr}^4}{M^4}\right)\right]$$

Discard events if Busoni, De Simone, Morgante, Riotto 1307.2253, +Gramling 1402.1275, +Jacques 1405.3101

$$Q > M \equiv \frac{M_*}{\sqrt{g_q g_\chi}}$$

✗ assume underlying model ⇒ study model-dependent

✗ cut procedure throws away highest energy events

? Is it possible to remain model-independent and use an EFT?